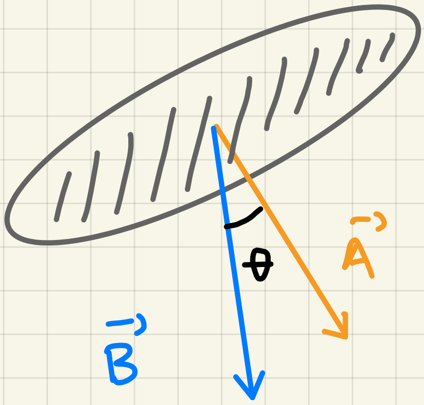
SEMINAR 5 – ELECTROMAGNETIC INDUCTION

General Remarks

The magnetic flux through a surface A is defined as $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$, where θ is the angle between the vectors $\vec{B}$ and $\vec{A}$.



Faraday's law states that an induced emf, $\mathcal{E}$, is present if there is a time variation of Φ_B:

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

Substituting $\Phi_B = BA\cos\theta$ gives,

$$\mathcal{E} = -\frac{d}{dt}\left(BA\cos\theta\right)$$

It can be seen that we can obtain an induced emf, or current (for closed circuits) if any of B, A or θ varies with time.

- In the following problems we study situations in which an induced emf and current are present due to the variation of the three quantities.

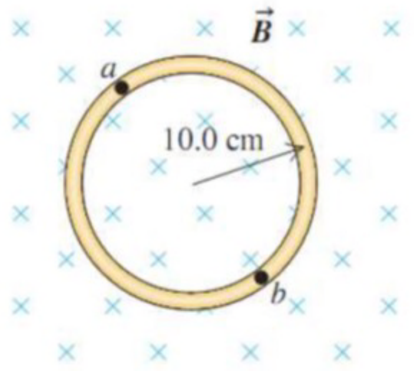
Lenz's law states that an induced current or emf always tends to <u>oppose</u> or <u>cancel</u> the change, in Φ_B, that caused it.

In practice, the induced current produces an induced field, $\vec{B}_{ind}$, which will be directed as follows:

(1) $\vec{B}_{ind}$ opposite to $\vec{B}$ if Φ_B increases

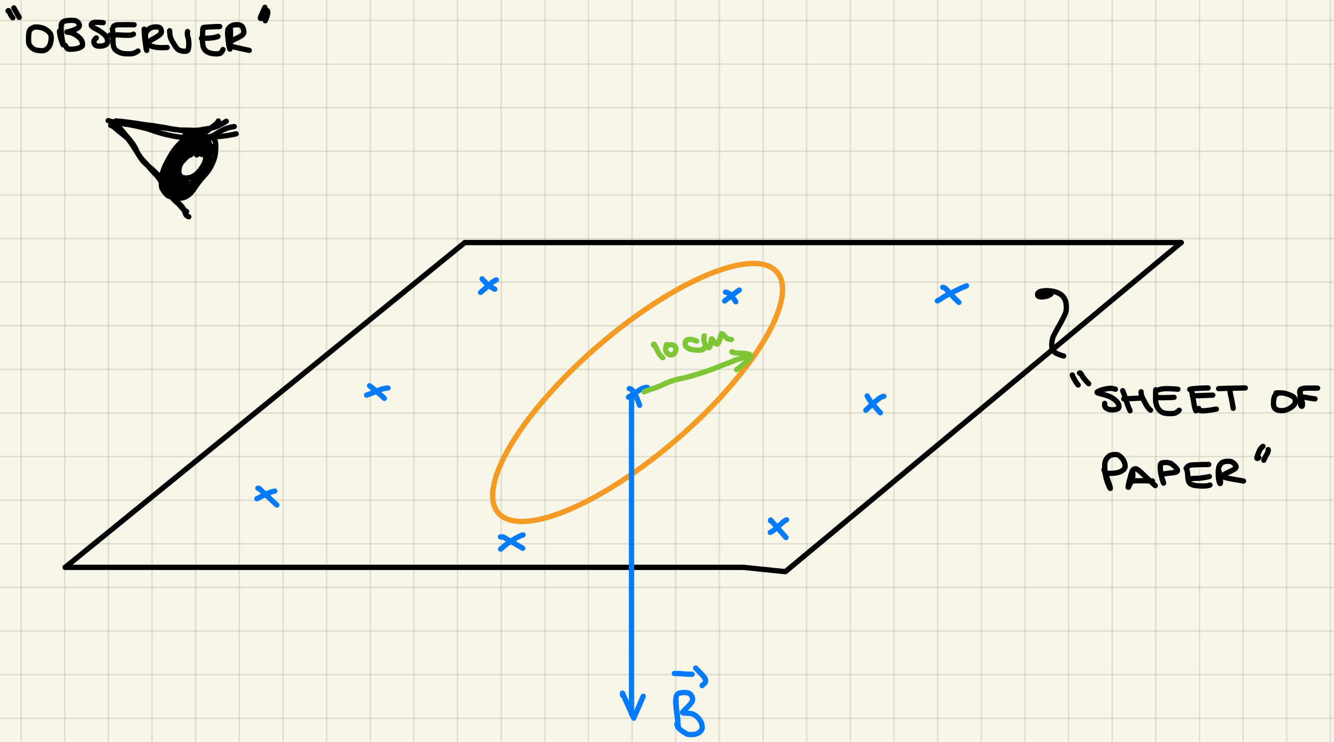
(2) $\vec{B}_{ind}$ the same as $\vec{B}$ if Φ_B decreases

1/ A circular loop of wire is in a region of spatially uniform magnetic field, as shown in Fig. below. The magnetic field is directed into the plane of the figure. Determine the direction (clockwise or counterclockwise) of the induced current in the loop when (a) B is increasing; (b) B is decreasing; (c) B is constant with value Explain your reasoning.

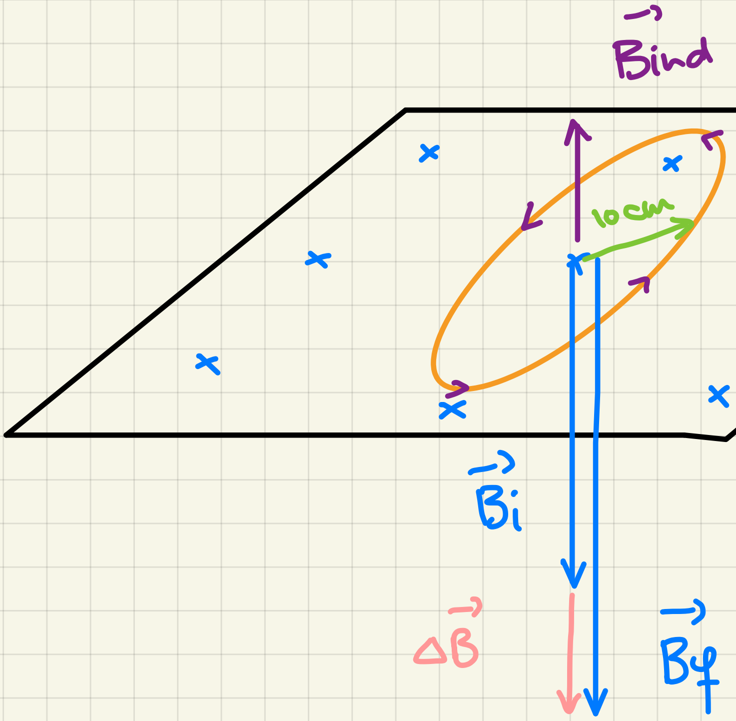


Observation:

The ⊗ sign means that the magnetic induction $\vec{B}$ is oriented perpendicular to the sheet of paper, pointing inwards, as represented below.



(a) B is increasing



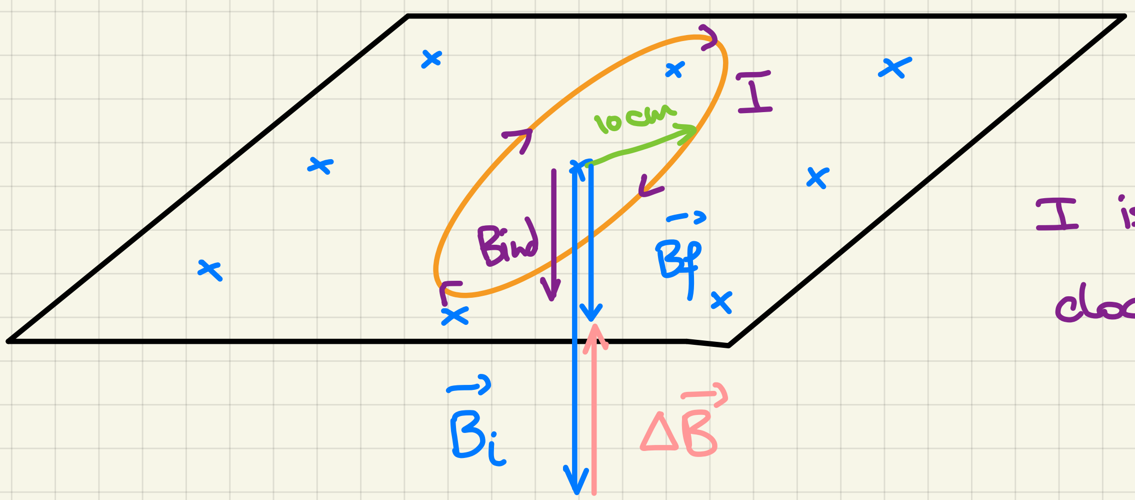
I is oriented counterclockwise

- if $\vec{B}$ increases $B_f > B_i$ and $\Delta\vec{B}$ is oriented as shown above

- if $\vec{B}$ increases, this means that the magnetic flux, Φ_B, increases $\Rightarrow$ there will be an <u>induced current</u>, I, in the loop (closed circuit)

- According to Lenz's law, the induced current is oriented in order to cancel the increase in Φ_B that caused it $\Rightarrow$ I will produce an induced field $\vec{B}_{ind}$, opposed to $\vec{B}$, in order to cancel the increase of Φ_B. For this, I will be directed counterclockwise.

(b) $\vec{B}$ is decreasing



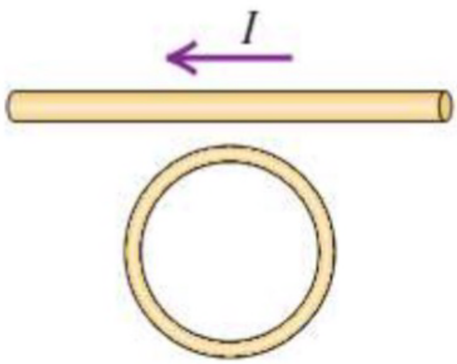
I is oriented clockwise

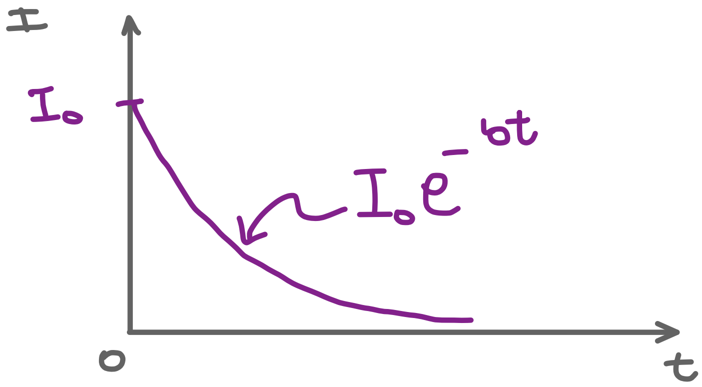
- $\vec{B}$ is decreasing, $B_f < B_i$, and $\Delta\vec{B}$ is oriented upwards, as shown above.

- $\Delta\vec{B}$ causes a decrease in the magnetic flux, Φ_B, through the loop $\Rightarrow$ there will be an induced current, I, through the Loop (closed circuit)

- According to Lenz's law the induced current, I, will have such a direction to cancel the decrease of the magnetic flux through the loop. This means that it will have to produce a $\vec{B}$ind that is oriented downwards $\Rightarrow$ I will circulate clockwise.

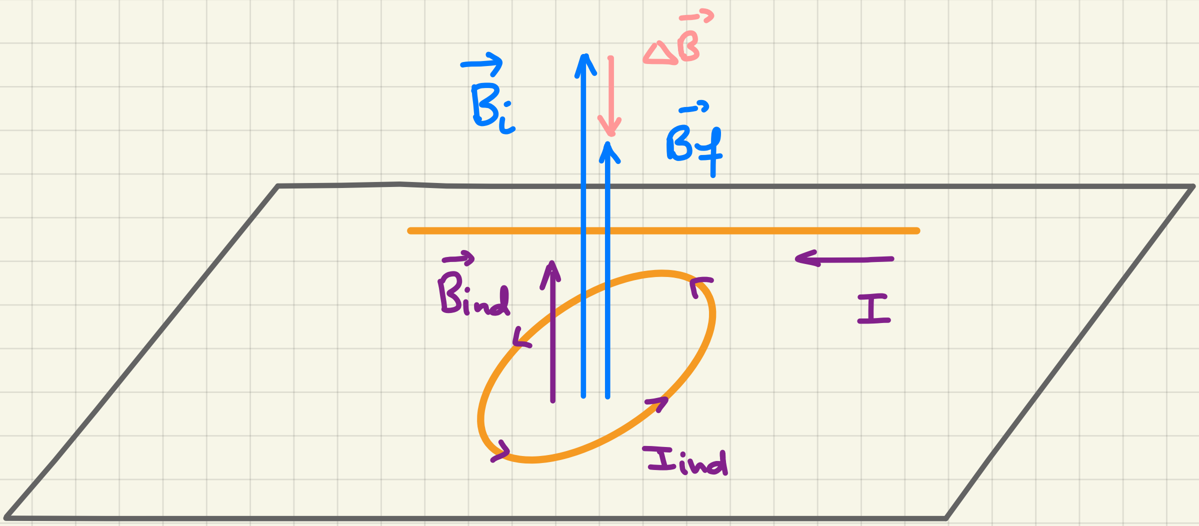
2/ The current in fig below obeys the equation: $I(t) = I_0 e^{-bt}$ where b>0. Find the direction (clockwise or counterclockwise) of the current induced in the rebound coil for t>0.



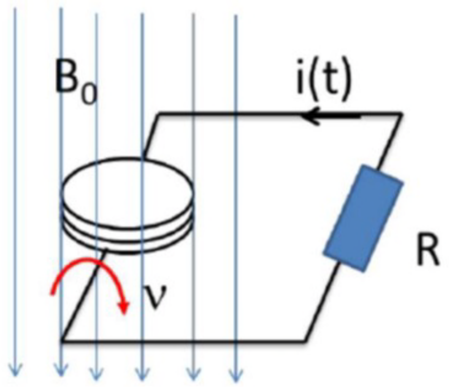


- It can be seen that I produces a magnetic field through the loop. The magnetic induction, $\vec{B}$ in the loop decreases in time, because $I = I_0 e^{-bt}$ decreases in time.
- The flux variation through the coil is thus due to the decrease of $\vec{B}$, $\Delta\vec{B}$. Φ_B decreases.
- The direction of the current needs to be oriented to produce a $\vec{B}_{ind}$ oriented in the same direction as $\vec{B}$ in order to cancel the decrease of Φ_B.

I_{ind} will circulate counterclockwise

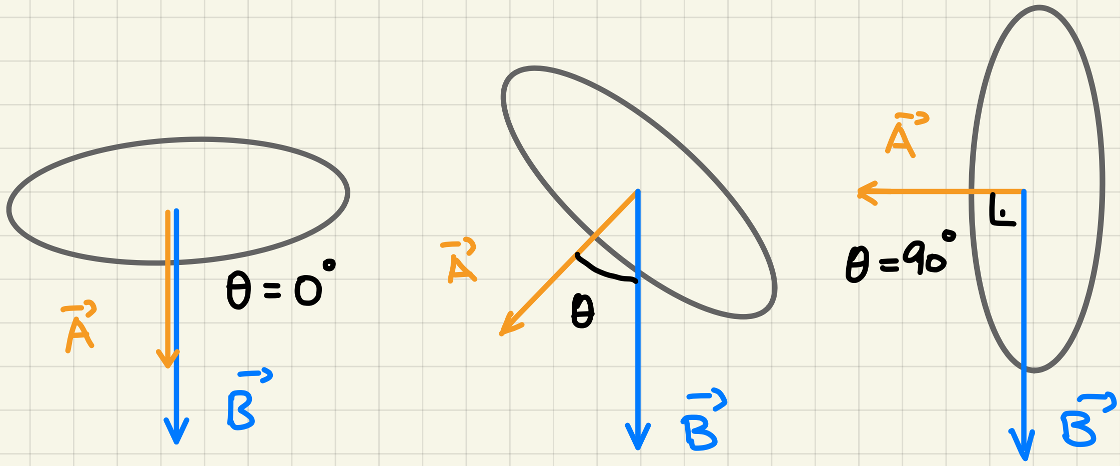


3/ A circular coil composed by N turns of radius r=1cm rotates in a constant magnetic field B=0.1T (see fig) with the frequency f=10Hz. Explain the origin of the induced electromotive voltage and calculate the current intensity in a resistor R=1kOhm connected in the circuit.



(Assume N = 100 turns)

- In this case $\vec{B_0}$ is constant and A is also constant, however the direction between $\vec{B}$ and $\vec{A}$ is varying, see below



- $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NBA\cos\theta)$$

N is introduced because we have to consider the flux change through each winding of the rotating coil.

- $\mathcal{E}$ is induced because the variation of Φ_B due to the changing θ.

We will consider $N = 100$

$\theta(t) = \omega t$, where ω is the angular velocity of the rotating coil.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(NBA\cos(\omega t)\right) =$$

$$= -NBA\frac{d}{dt}\left(\cos(\omega t)\right) = -NBA\left(-\sin(\omega t)\right)\cdot\omega$$

$$= NBA\omega\sin(\omega t)$$

$$\omega = 2\pi f = 2\cdot\pi\cdot 100\,\text{Hz} = 628{,}319\ \text{rad/s}$$

$$A = \pi r^2 = \pi\cdot\left(10^{-2}\,\text{m}\right)^2 = 3{,}142\ 10^{-4}\,\text{m}^2$$

$$\mathcal{E}(t) = (100\,\text{turns})(0{,}1\,\text{T})\left(3{,}142\cdot 10^{-4}\,\text{m}^2\right)\cdot$$

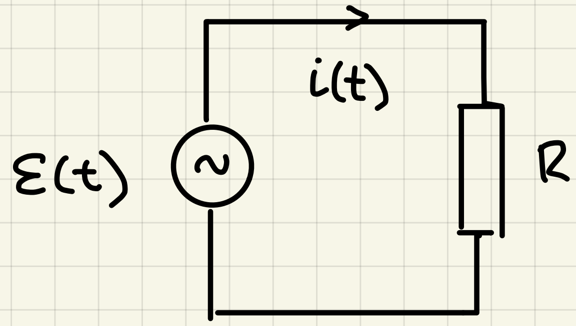
$$\cdot\left(628{,}319\,\frac{\text{rad}}{\text{s}}\right)\sin\left(628{,}319\,\frac{\text{rad}}{\text{s}}\cdot t\right)$$

$$\mathcal{E}(t) = 1{,}974\sin(628{,}319\,t)\ (\text{V})$$

$$\mathcal{E}(t) \simeq 2\sin(628{,}3t)\,(\text{V})$$

- $\mathcal{E}(t)$ is alternative because of the fact that the flux variation is periodic. As the coil rotates, Φ_B increases and decreases as a function of the relative orientation between $\vec{B}$ and $\vec{A}$.

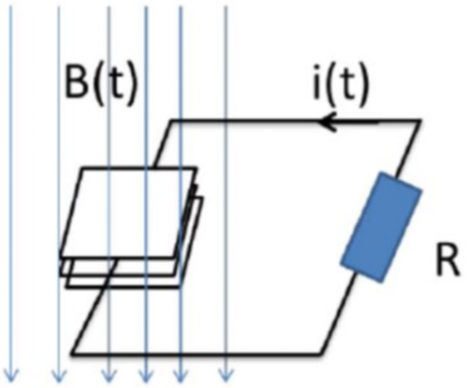
- In order to find the current intensity through R, we can replace the rotating coil by an alternative voltage source. The equivalent circuit will be:

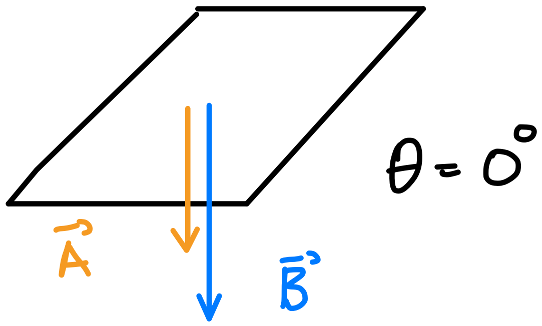


Using Ohm's law: $i(t) = \dfrac{\mathcal{E}(t)}{R} = \dfrac{2\sin(628{,}3t)\ \text{V}}{10^3\ \Omega} \Leftrightarrow$

$\Leftrightarrow\ i(t) = 2\cdot 10^{-3} \sin(628{,}3t)\ (\text{A})$.

4/ A square coil having the lateral side of 3cm composed by N turns is placed in a perpendicular to the surface time varying magnetic field $B = 0.1\sin(200\pi t + \pi/3)$. Calculate the intensity of the induced current in the circuit if the resistor is R=1kOhm. (Assume N= 100 turns)





The flux variation through the coil is due to the periodic variation of the magnitude of the magnetic induction, $\vec{B}(t)$.

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(N B(t) A \cos\theta\right) \overset{\theta=0°}{=}$$

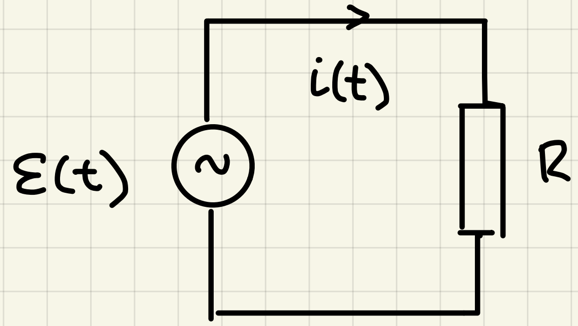
$$= -\frac{d}{dt}\left(N B(t) A\right) = -NA\frac{dB(t)}{dt} =$$

$$= -(100 \text{ turns})(9\cdot 10^{-4} m^2)\frac{d}{dt}\left(0.1\sin\left(200\pi t + \frac{\pi}{3}\right)\right)$$

$$= -9\cdot 10^{-2}\cdot\left(0{,}1\cos\left(200\pi t + \pi/3\right)\right)\cdot(200\pi) =$$

$$= -1{,}8\pi\cos\left(200\pi t + \pi/3\right)\ (V)$$

Like in the previous problem we can replace the coil with an alternative voltage source.



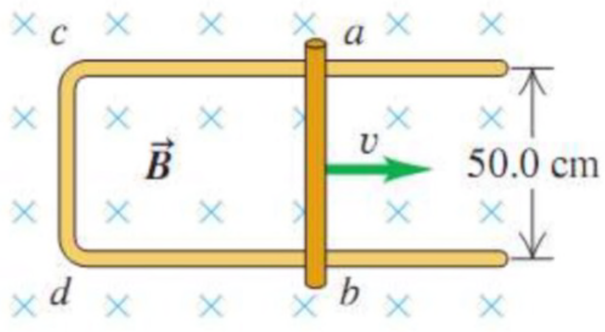
Using Ohm's law:

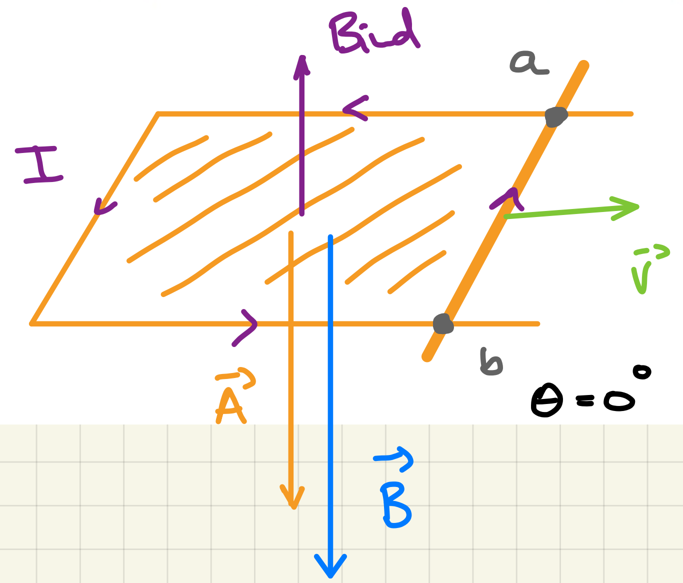
$$i(t) = \frac{\mathcal{E}(t)}{R} =$$

$$= \frac{-1{,}8\pi}{10^3} \sin\left(200\pi t + \frac{\pi}{3}\right)$$

$$\Leftrightarrow \quad i(t) \simeq -5{,}7 \cdot 10^{-3} \sin\left(200\pi t + \pi/3\right) \ (A)$$

5/ The conducting rod *ab* shown in Fig. below makes contact with metal rails *ca* and *db* . The apparatus is in a uniform magnetic field of 0.800 T, perpendicular to the plane of the figure (a) Find the magnitude of the emf induced in the rod when it is moving toward the right with a speed v=7.5m/s. (b) In what direction does the current flow in the rod? (c) If the resistance of the circuit *abdc* is 1.5Ohm (assumed to be constant), find the force (magnitude and direction) required to keep the rod moving to the right with a constant speed of 7.5m/s. You can ignore friction. (d) Compare the rate at which mechanical work is done by the force (Fv) with the rate at which thermal energy is developed in the circuit (I^2R).

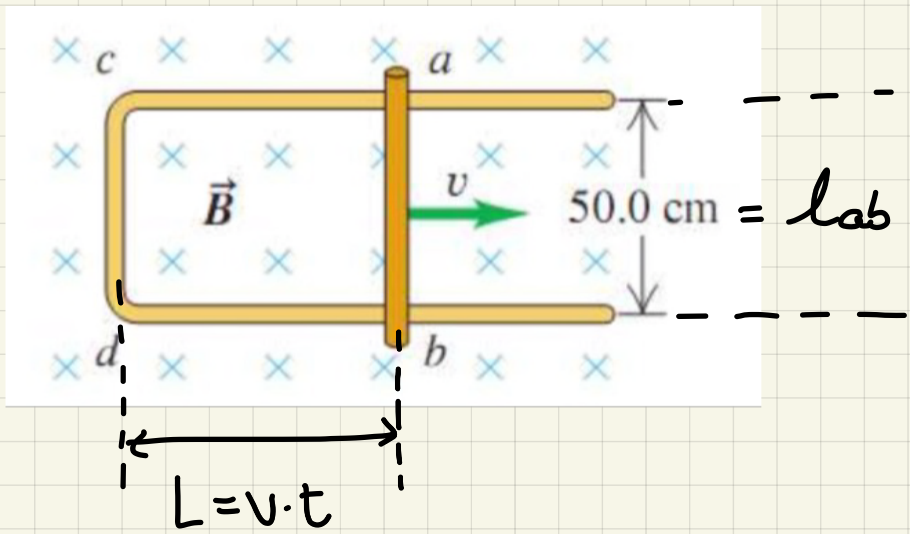




- In this case the flux, $\Phi_B = BA\cos\theta$, varies because the area A increases as the rod ab moves. Therefore $\frac{d\Phi_B}{dt} > 0$, the flux increases.

- The induced emf, ε, and the corresponding current intensity, I, must oppose the flux increase, so I will have to create an induced magnetic field that opposes $\vec{B}$, to cancel the increase of Φ_B.

(a)



The area A of the (abcd) rectangle is time dependent:

$$A(t) = L \cdot l_{ab} = (v \cdot t) \cdot l_{ab} = v\, l_{ab} \cdot t$$

According to Faraday's Law:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(B\,A(t)\cos 0^\circ\right) =$$

$$= -\frac{d}{dt}\left(B \cdot v \cdot l_{ab} \cdot t\right) = -B\,v\,l_{ab}$$

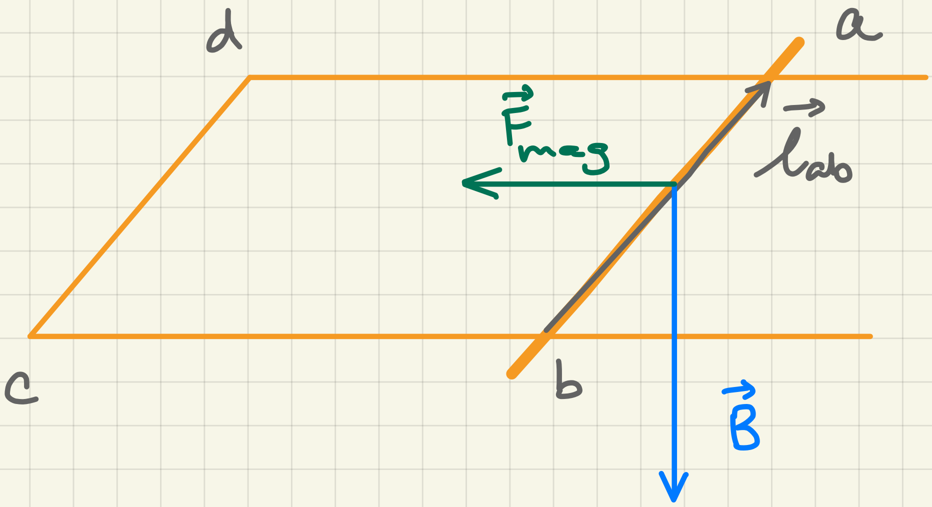
$$= -(0{,}8\,T)(7{,}5\,m/s)(0{,}5\,m) =$$

$$= -3\,V \Rightarrow |\mathcal{E}| = 3\,V$$

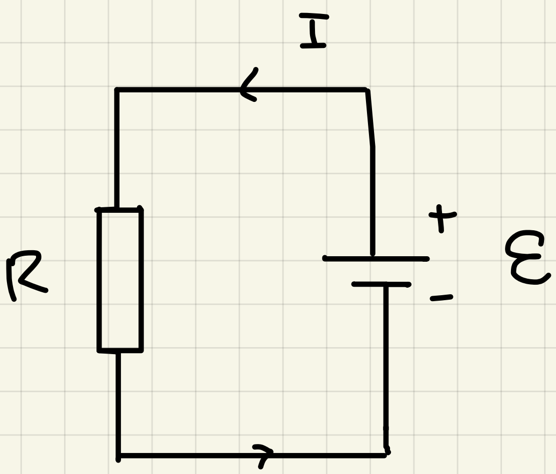
(b) As it can be seen on the previous page the induced current will circulate counterclockwise in order to oppose the magnetic flux increase.

(c) Because the rod ab is moving in a magnetic field, and there is the current I passes through it, there will be a magnetic force, $\vec{F}_{mag}$, acting on it.

$$\vec{F}_{mag} = I\left(\vec{l}_{ab} \times \vec{B}\right)$$



in order to find I, we use Ohm's law in the equivalent electrical circuit in which the (ab) rod acts as a voltage source.



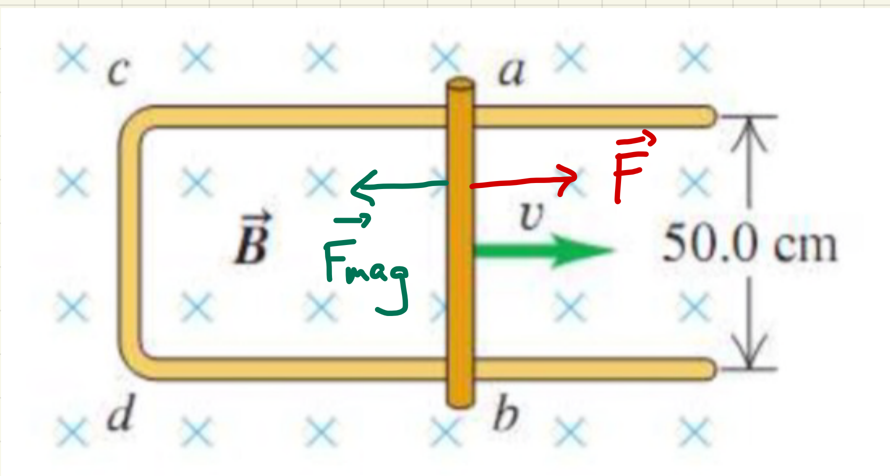
$$I = \frac{\mathcal{E}}{R} = \frac{3V}{1{,}5\Omega} = 2A$$

Since $\vec{B} \perp \vec{l}_{ab}$

$$F_{mag} = I\, l_{ab} B \sin 90^\circ = (2A)\cdot(0{,}5m)\cdot(0{,}8T) =$$
$$= 0{,}8\,N$$

However, you can observe that $\vec{F}_{mag}$ is oriented in the opposite direction as $\vec{v}$, the velocity of the rod. if $\vec{F}_{mag}$ were the only force acting on the rod, it would produce an acceleration opposite to $\vec{v}$, which would bring the rod to a stop.

if the rod travels at a constant velocity it means that there is an additional force, $\vec{F}$, equal and opposite to $\vec{F}_{mag}$, so that the total force acting on the rod is 0.



$\vec{v} = \text{const} \Rightarrow$

$\vec{F} + \vec{F}_{mag} = 0 \Leftrightarrow$

$\vec{F} = -\vec{F}_{mag}$

$F = F_{mag} = 0{,}8\,N$

(d) The power, P, developed by $\vec{F}$, which is the rate at which F is performing work on the rod is:

$$P = F \cdot v = (0.8\ N) \cdot (7{,}5\ m/s) = 6\ W$$

The thermal energy developed in the circuit:

$$P' = I^2 R = (2A)^2 \cdot (1{,}5\ \Omega) = 6W$$

This is an important result because it shows that in order to setup a current in the circuit and therefore to produce energy, we have to perform work on the system.